A Dynamic Model of Airline Competition

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Abstract

We develop a dynamic model of collusion in airport-pair routes for selected US airlines and specify the first order conditions using a state-space representation that is estimated by Kalman-filtering techniques using the Databank 1A (DB1A) Department of Transportation (DOT) data during the period 1979I-1988IV. We consider two airlines, American (AA) and United (UA) and four city-pairs. Our measure of market power is based on the shadow value of long-run profits in a two-person strategic dynamic game and we find evidence of relative market power of AA in three of the four city pairs we analyze.

1 Introduction

In this paper, we develop a dynamic model of collusion in airport-pair routes for selected U.S. airlines. In earlier work on collusion and market power, Roeller and Sickles (2000) estimated a two-stage static structural model in which the firms play a repeated sequence of one-shot capacity and pricing games. They found that the market conduct parameter, whose value can differentiate among Bertrand, Cournot-Nash and monopolistic equilibrium, had adjusted to a value closer to a competitive equilibrium as the industry was deregulated. Using a somewhat different time series approach, Alam and Sickles (2000) looked at market conduct on the supply side (focusing on the degree of inefficiency) and found that a similar convergence to competitive equilibrium took place in the U.S. airline industry after its deregulation in 1976. Captain and Sickles (1997) utilized a one-stage static structural model of market conduct for the European airlines in which labor choices were endogenous and where firms play a pricing game and estimated conduct to be between a competitive and Betrand solution. Unfortunately, these studies relying on the

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conjectural variations approach are myopic to the past and are not forward-looking when current actions are considered.

The models we specify and estimate in this paper are based on the realistic assumption that firms consider the future when they make a current decision and that decisions they make today will influence outcomes in the future. In this sense, it is similar in spirit to the work of Captain *et al* (2007) although they did not estimate but rather calibrated their model. In this paper, we consider models that also allows for more flexibility in describing how equilibrium outcomes can be characterized. Solutions are not necessarily at the nodes of the solutions identified by the static market conduct approaches. As pointed out in Perloff *et al* (2007), dynamic strategic considerations also often require new methods of estimation, such as the state-space methods which we employ herein, instead of the standard nonlinear least squares. Specifically, our paper develops a dynamic model of collusion in airport-pair routes for selected US airlines and specifies the first order conditions based on a state-space representation that is estimated by Kalman-filtering techniques (1960). Our model controls for economy-wide exogenous variables and city-pair specific variables. We examine two U.S. commercial airline firms, United and American Airlines between 1979 Q1 and 1988 Q4.

After the Airline deregulation Act of 1978, the airline industry moved from service-based to price-based competition. Carriers were able to set their own fares, select and drop routes, and control flight frequency. U.S. airlines continue to face substantial upheavals in the form of mergers, failures, bankruptcy filings, reorganizations and operating losses. As the concentration of the industry continues to increase and as the number of profitable incumbents continues to dwindle, as fuel prices continue to soar, the survivability and prosperity of incumbents becomes increasingly problematic. In this economic and institutional setting, the dynamics of strategic decision making involving various forms of collusion in the form of formal alliances are important to understand. Another empirically attractive feature of this industry, a consequence of the strict filing requirements imposed by the federal government, is the wealth of accessible data not generally available in most other industries.

The paper is structured as follows. The motivation for analyzing market power in a dynamic setting in the airline industry is provided in section 2. Section 3 introduces our econometric model and derives important measures of market power in a dynamic strategic setting, the shadow price of profits. Section 4 discusses our empirical illustration and results. Section 5 provides concluding remarks.

2 Market power and the dynamics of firm interaction

2.1 Rationale for collusion and market power

Standard economic theory predicts that under mild assumptions when there are only a few agents on one side of a market those agents will often possess market power – the ability to alter profitably prices away from competitive levels without losing all customers to competitors. The Cournot model of duopoly assumes that a firm never had to consider the reaction of its competitor to its price or quantity choice. In the Bertrand model, a firm could undercut its rival's price at the margin and compete-away all of the rival's customers. In practice, however, a firm may recognize that if it undercut its rival, the rival will respond by cutting its own price, ultimately leading to a short-run gain in sales but a long-run reduction in the price level.

Consider a dynamic model in which these concerns arise. Each firm i attempts to maximize the discounted value of profits, $\Sigma_{t=1}^{\infty}\beta^{t-1}\pi_{it}$, where π_{it} is firm i's profit in period t. If each firm initially charges p^m, the monopoly price, then industry profit is maximized. It continues to charge p^m in period t if in every period preceding t, both firms have charged p^m; otherwise it sets its price at marginal cost c forever. This is equilibrium if the discount factor is sufficient high. In charging p^m, the firm earns half the monopoly profit in each period. By deviating from this price, a firm can earn maximum profit, Π^m, during the period of deviation but it receives zero forever. Therefore, if $\frac{\Pi^m}{2}(1+\beta+\beta^2+...)\geq\Pi^m$, if $\beta\geq\frac{1}{2}$, then these strategies are equilibrium ones, which are also called trigger strategies. There are many other equilibria in this game, any price between the competitive price and the monopoly price can be sustained as an equilibrium price as long as the discount factor is greater than $\frac{1}{2}$. The Folk Theorem summarizes this outcome: in an infinite repeated game, any feasible discounted payoffs that give each player, on a per-period basis, more than the lowest payoff that he could guarantee himself in a single play of the simultaneous-move component game can be sustained as the payoffs of an subgame perfect Nash equilibrium if players discount the future to a sufficiently small degree.

2.2 Rationale for dynamic decision making

Beginning with the classic work of Chamberlin (1929), researchers have continued to explore the implications of repeated interaction between collusive oligopolists, as well as factors that may hinder such collusion in repeated pricing games. Consider a small number of identical firms producing a homogeneous product. Chamberlin conjectured that the firms in the industry would charge the monopoly price. Each firm makes profit Π^m/n, where $\Pi^m\equiv\Pi(p^m)$. As Chamberlin noted, detection lags and asymmetries between firms are two factors that may hinder collusion. Tacit collusion is enforced by the threat of retaliation. But retaliation can occur only when it is learnt that some member of the industry has deviated. For example, before the existence of online travel companies, the prices charged by airlines may be somewhat hidden. However, in the current environment, and to some degree in the environment that existed during our sample period, the prices charged by an airline can be observed fairly quickly by its competitors.

Oligopolists are likely to recognize that one threat to collusion is lack of secrecy and consequently may take steps to control it. An example is Orbitz, which is an online travel company funded by five airlines, American, Continental, Delta, Northwest and United. Under asymmetric conditions, the oligopolists' marginal costs may differ, thus they have different monopoly prices. Low-cost firms would prefer to coordinate on a lower price than the higher-cost firms. Theory suggests that as an industry becomes more competitive, it becomes more important for a firm to perform efficiently relative to other firms if it is going to survive. This is one of the sources of dynamic productive efficiency revealed in the U.S. airline industry after deregulation by Alam and Sickles (2000). But, how does market power for airlines arise? Do they arise from barriers to entry, from sunk costs of gate and slot access, scale and network economies, or from hub-and-spoke systems which can give carriers market power even on relatively competitive routes? Borenstein (1989), among others, has estimated the importance of route and airport dominance in determining the degree of market power exercised by an airline. His results indicate that an airline's share of passengers on a route and at the endpoint airports significantly influence its ability to mark up price above cost. The high markups of a dominant airline, however, do not create much of an "umbrella" effect from which carriers with smaller operations in the same markets can benefit. Other rationale for market power on routes come from Berry (1992) who pointed out that airline firms are suited to serve different routes by virtue of unobserved heterogeneities (market niches) that allow them to exploit monopoly power over their differentiated product. An alternative view is found in the work of Morrison and Winston (2000) who examine merger activity and the factors that influence them for the U.S. airlines. They make the empirical argument that mergers are not driven by a desire to obtain market power but rather by the acquiring carriers' desire to expand their international routes. These routes tend to be more profitable on average than domestic routes because of bilateral agreements that limit entry. Moreover, the acquired carriers often have strong incentives to merge because of poor financial prospects (Crandall and Winston, 2006). There is a substantial body of literature that has examined the reasons for and against the presence of market power in the commercial airline industry. What we consider below is an econometric model that can provide evidence for or against such market power in a dynamic setting of strategically interacting carriers at a level of disaggregation that provides us the best empirical measures of such potential conduct, which is the city pair route.

3 Specification of the dynamic Model

We look at a set of city pairs in which American and United are dominant firms and assume that the competitive environment in which the firms compete is Markovian. The source of data was the Databank 1A (DB1A) of the Department of Transportation (DOT). The data are taken directly from airline ticket stubs, along with the total ticket price; the data contains such information as the carrier, origin, destination and class for all trip segments. The data are by airline and by quarter from 1979 Q1 to 1988 Q4. The airlines in the study are American and United. Economy-wide exogenous variables and city pair specific variables are important controls in the model and consist of; the existence of a frequent flier program, strikes, alliances, the Gulf War, the air controller's strike under the Reagan Administration, mergers, price of jet fuel and national taxes on airlines. The city-pair specific exogenous variables consist of city pair and region dummies, international airport dummies, delays, entry and exit, share of minor carriers on the route, unemployment rate for each city, percent change in state GDP and relative or actual marginal cost. These variables have been examined in a somewhat different reduced form market power setting in Perloff, *et al* (2003) and in the context of constructing a hedonic airline price index (Good, *et al*, 2007).

Turning to the model, AA (Firm1)'s dynamic program is

$$J^1(q^1_{c,t-1}, q^2_{c,t-1}, x_t, z_t) = \max\, p^1_{c,t} q^1_{c,t} - c^1(q^1_{c,t}) + \beta J^1(q^1_{c,t}, q^2_{c,t}, x_{t+1}, z_{c,t+1})$$

on choosing output $q^1_{c,t}$. While for UA (Firm2), its dynamic program is

$$J^2(q^1_{c,t-1}, q^2_{c,t-1}, x_t, z_t) = \max\ p^2_{c,t}q^2_{c,t} - c^2(q^2_{c,t}) + \beta J^2(q^1_{c,t}, q^2_{c,t}, x_{t+1}, z_{c,t+1})$$

on choosing output $q^2_{c,t}$, where β is the discount factor, the c subscript refers to an airport pair, the t subscript is the time period, x_t are economy-wide exogenous variables, and $z_{c,t}$ are city-pair specific exogenous variables. The parameter β is at 0.90 in our analysis below.

There are a variety of ways that we can allow the exogenous variables to influence the shadow value of $q^1_{c,t}$. We outline three different methods. First, we can write the first-order condition for AA (Firm1) as

$$p^1_{c,t} + q^1_{c,t}\frac{\partial p^1_{c,t}}{\partial q^1_{c,t}} - \frac{\partial c^1}{\partial q^1_{c,t}} + \alpha^1_{c,t}D_c + \beta(\lambda^{11}_t q^1_{c,t} + \lambda^{12}_t q^2_{c,t} + \lambda^{13}_t x^1_{t+1} + \lambda^{14}_t z^1_{t+1}) = 0$$

and the first-order condition for UA (Firm2) as

$$p^2_{c,t} + q^2_{c,t}\frac{\partial p^2_{c,t}}{\partial q^2_{c,t}} - \frac{\partial c^2}{\partial q^2_{c,t}} + \alpha^2_{c,t}D_c + \beta(\lambda^{21}_t q^1_{c,t} + \lambda^{22}_t q^2_{c,t} + \lambda^{23}_t x^2_{t+1} + \lambda^{24}_t z^2_{t+1}) = 0$$

where D_c is a dummy for the particular city pair c. The above two equations are the measurement equations which will be used in the Kalman filters (see the Appendix for a more complete discussion of how the Kalman filter is set up for this problem). The shadow value of $q^1_{c,t}$ (see the Appendix for a detailed discussion of how the shadow price is constructed for our dynamic program) is

$$\lambda^{11}_t q^1_{c,t} + \lambda^{12}_t q^2_{c,t} + \lambda^{13}_t x^1_{t+1} + \lambda^{14}_t z^1_{t+1}$$

and indicates the extent of market power for AA while the shadow value of $q^2_{c,t}$ is

$$\lambda^{21}_t q^1_{c,t} + \lambda^{22}_t q^2_{c,t} + \lambda^{23}_t x^2_{t+1} + \lambda^{24}_t z^2_{t+1}$$

and provides us a similar measure of market power for UA.

The Kalman filter for AA (Firm1) is

$$p^1_{c,t} + q^1_{c,t}\frac{\partial p^1_{c,t}}{\partial q^1_{c,t}} - \frac{\partial c^1}{\partial q^1_{c,t}} + \alpha^1_{c,t}D_c + \beta(\lambda^{11}_t q^1_{c,t} + \lambda^{12}_t q^2_{c,t} + \lambda^{13}_t x^1_{t+1} + \lambda^{14}_t z^1_{t+1}) = 0 \Leftrightarrow$$

$$(\frac{\partial c^1}{\partial q^1_{c,t}} - p^1_{c,t} - q^1_{c,t}\frac{\partial p^1_{c,t}}{\partial q^1_{c,t}})\beta^{-1} = \alpha^1_{c,t}\frac{D_c}{\beta} + \lambda^{11}_t q^1_{c,t} + \lambda^{12}_t q^2_{c,t} + \lambda^{13}_t x^1_{t+1} + \lambda^{14}_t z^1_{t+1} \Leftrightarrow$$
$$Y^1_t = X^{1'}_t B^1_t + e_t$$

where

$$Y^1_t = (\frac{\partial c^1}{\partial q^1_{c,t}} - p^1_{c,t} - q^1_{c,t}\frac{\partial p^1_{c,t}}{\partial q^1_{c,t}})\beta^{-1}$$
$$X^{1'}_t = (\frac{D_c}{\beta}, q^1_{c,t}, q^2_{c,t}, x^1_{t+1}, z^1_{t+1})$$
$$B^{1'}_t = (\alpha^1_{c,t}, \lambda^{11}_t, \lambda^{12}_t, \lambda^{13}_t, \lambda^{14}_t)$$

while for UA (Firm2) the Kalman filter is

$$p^2_{c,t} + q^2_{c,t}\frac{\partial p^2_{c,t}}{\partial q^2_{c,t}} - \frac{\partial c^2}{\partial q^2_{c,t}} + \alpha^2_{c,t}D_c + \beta(\lambda^{21}_t q^1_{c,t} + \lambda^{22}_t q^2_{c,t} + \lambda^{23}_t x^2_{t+1} + \lambda^{24}_t z^2_{t+1}) = 0 \Leftrightarrow$$
$$(\frac{\partial c^2}{\partial q^2_{c,t}} - p^2_{c,t} - q^2_{c,t}\frac{\partial p^2_{c,t}}{\partial q^2_{c,t}})\beta^{-1} = \alpha^2_{c,t}\frac{D_c}{\beta} + \lambda^{21}_t q^1_{c,t} + \lambda^{22}_t q^2_{c,t} + \lambda^{23}_t x^2_{t+1} + \lambda^{24}_t z^2_{t+1} \Leftrightarrow$$
$$Y^2_t = X^{2'}_t B^2_t + e_t$$

where

$$Y^2_t = (\frac{\partial c^2}{\partial q^2_{c,t}} - p^2_{c,t} - q^2_{c,t}\frac{\partial p^2_{c,t}}{\partial q^2_{c,t}})\beta^{-1}$$
$$X^{2'}_t = (\frac{D_c}{\beta}, q^1_{c,t}, q^2_{c,t}, x^2_{t+1}, z^2_{t+1})$$
$$B^{2'}_t = (\alpha^2_{c,t}, \lambda^{21}_t, \lambda^{22}_t, \lambda^{23}_t, \lambda^{24}_t)$$

The state equations are not defined. One possibility is a simple auto-regression for both carriers such as

$$B^1_t = B^1_{t-1} + \upsilon^1_t$$
$$B^2_t = B^2_{t-1} + \upsilon^2_t$$

while another possibility is to include the strictly exogenous variables.

Second, we can use the same approach as above, but assume that the coefficients on the x and z terms are constants (no t subscripts): λ^3 and λ^4. Now we only have to determine two terms using Kalman filters. Third, we can reduce the number of λ terms. For AA (Firm1) we can write

$$p_{c,t}^1 + q_{c,t}^1 \frac{\partial p_{c,t}^1}{\partial q_{c,t}^1} - \frac{\partial c^1}{\partial q_{c,t}^1} + \alpha_{c,t}^1 D_c + \beta\left[\lambda_t^{11} q_{c,t}^1 + \lambda_t^{12} q_{c,t}^2 + \lambda_t^{13} f(x_{t+1}^1, z_{t+1}^1)\right] = 0$$

and for UA (Firm2) we can write

$$p_{c,t}^2 + q_{c,t}^2 \frac{\partial p_{c,t}^2}{\partial q_{c,t}^2} - \frac{\partial c^2}{\partial q_{c,t}^2} + \alpha_{c,t}^2 D_c + \beta\left[\lambda_t^{21} q_{c,t}^1 + \lambda_t^{22} q_{c,t}^2 + \lambda_t^{23} f(x_{t+1}^2, z_{t+1}^2)\right] = 0$$

We now have three λ terms that we need to determine using Kalman filters. We could allow λ_t^{ij} to be determined by a Kalman filter that depends on x and not on z. The state equations are then

$$\begin{aligned}
\lambda_t^{11} &= \rho_0^{11} + \rho_1^{11}\lambda_{t-1}^{11} + \rho_2^{11}x_{t+1}^1 + \rho_3^{11}q_t^1 + \rho_4^{11}q_t^2 + \varepsilon_t \\
\lambda_t^{12} &= \rho_0^{12} + \rho_1^{12}\lambda_{t-1}^{12} + \rho_2^{12}x_{t+1}^1 + \rho_3^{12}q_t^1 + \rho_4^{12}q_t^2 + \varepsilon_t \\
\lambda_t^{21} &= \rho_0^{21} + \rho_1^{21}\lambda_{t-1}^{21} + \rho_2^{21}x_{t+1}^2 + \rho_3^{21}q_t^1 + \rho_4^{21}q_t^2 + \varepsilon_t \\
\lambda_t^{22} &= \rho_0^{22} + \rho_1^{22}\lambda_{t-1}^{22} + \rho_2^{22}x_{t+1}^2 + \rho_3^{22}q_t^1 + \rho_4^{22}q_t^2 + \varepsilon_t
\end{aligned}$$

Providing that f is linear in z and providing that we know β, the first order condition is linear in parameters and can be estimated using the linear Kalman Filter.

4 Estimation results

The estimation results reported below are based on the first method discussed above. The state equation is defined as a simple auto-regression. Since we estimate the model by each city pair, D_c is dropped. We include D_c when we compare AA with UA by multiple city pairs.

We illustrate in detail how we derive the estimated results of AA between city-pair Chicago (ORD) and San Diego (SAN). Estimates for other city-pairs are straightforward extensions. The first-order condition for AA (Firm 1) between city pair Chicago (ORD) and San Diego (SAN) is:

$$\begin{aligned}
&p_t^1 + q_t^1 \frac{\partial p_t^1}{\partial q_t^1} - \frac{\partial c^1}{\partial q_t^1} + \alpha + \beta(\lambda_t^{11} q_t^1 + \lambda_t^{12} q_t^2 + \lambda_t^{13} x_{t+1}^1 + \lambda_t^{14} z_{t+1}^1) = 0 \Leftrightarrow \\
&(\frac{\partial c^1}{\partial q_t^1} - p_t^1 - q_t^1 \frac{\partial p_t^1}{\partial q_t^1})\beta^{-1} = \alpha\beta^{-1} + \lambda_t^{11} q_{c,t}^1 + \lambda_t^{12} q_{c,t}^2 + \lambda_t^{13} x_{t+1}^1 + \lambda_t^{14} z_{t+1}^1 \Leftrightarrow \\
&Y_t^1 = X_t^{1'} B_t^1 + e_t
\end{aligned}$$

where

$$Y_t^1 = (\frac{\partial c^1}{\partial q_t^1} - p_t^1 - q_t^1 \frac{\partial p_t^1}{\partial q_t^1})\beta^{-1}$$

$$X_t^{1'} = (\frac{1}{\beta}, q_t^1, q_t^2, x_{t+1}^1, z_{t+1}^1)$$

$$B_t^{1'} = (\alpha, \lambda_t^{11}, \lambda_t^{12}, \lambda_t^{13}, \lambda_t^{14})$$

We assume a simple demand function given by

$$P = A - Q = A - q_1 - q_2$$

where A is a constant.

We use estimates of the average cost when route is defined by city pair (**avcstcty**), average price when the route is a city pair (**avgprcty**), and the number of passengers on a route (**rtpass**) from Perloff *et al* (2003). Our results are based on an assumed discount factor β of 0.9, but a value of 0.95 was also examined and the results were qualitatively similar. This yields the following equation

$$Y_t^1 = (\frac{\partial c^1}{\partial q_t^1} - p_t^1 - q_t^1 \frac{\partial p_t^1}{\partial q_t^1})\beta^{-1} = (avctscty - avgprcty + rtpass)/0.9$$

We use national per capita income (**pcinc**) as an economy-wide exogenous variable X_t and Herfindahl index for city pair (**herfcty**) as an city-pair specific exogenous variable Z_t which appears to be the best city-pair specific exogenous variable we could find in the available data. In this context then

$$X_t^{1'} = (\frac{1}{\beta}, q_t^1, q_t^2, x_{t+1}^1, z_{t+1}^1) = (\frac{1}{0.9}, rtpass_t^1, rtpass_t^2, pcinc_{t+1}^1, h\text{erf}\, cty_{t+1}^1)$$

$$B_t^{1'} = (\alpha, \lambda_t^{11}, q_t^{12}, \lambda_t^{13}, \lambda_t^{14})$$

Estimation results are reported in Tables 1 and 2. The cities are Chicago (ORD), Salt Lake City (SLC), San Diego (SAN), San Francisco (SFO) and Seattle (SEA). Numerical issues with our algorithm prevented us from estimating accurately the standard errors for several of the variables. Those entries are left blank in the Tables.

City Pair	ORD-SAN	ORD-SEA	ORD-SFO	ORD-SLC
λ^{11} Estimates	0.64976652	0.34198363	0.35672006	0.00000596
λ^{11} SE	0.09058998	0.08449455	0.09268303	0.23502357
λ^{12} Estimates	0.00000552	0.16309741	0.16265314	0.44189663
λ^{12} SE	**	0.09752401	0.11300235	0.13558679
λ^{13} Estimates	0.13236929	0.15559350	0.14523205	0.24318685
λ^{13} SE	0.08278507	0.07760735	0.07974509	0.19779975
λ^{14} Estimates	0.00001244	0.00000927	0.00000201	0.00000029
λ^{14} SE	0.05990846	0.04294906	0.04234850	0.07371905

Table 1: AA

City Pair	ORD-SAN	ORD-SEA	ORD-SFO	ORD-SLC
λ^{21} Estimates	0.69631873	0.00034146	0.00017537	0.00000281
λ^{21} SE	0.10723028	0.35834513	**	**
λ^{22} Estimates	0.00001757	0.30445244	0.29264571	0.45626616
λ^{22} SE	0.19210776	0.11489267	0.09255699	0.12926968
λ^{23} Estimates	0.18620294	0.35685817	0.38122657	0.33791960
λ^{23} SE	0.11633113	0.10459605	0.08599149	0.04786982
λ^{24} Estimates	0.00000065	0.00000525	0.00000102	0.00001359
λ^{24} SE	0.1357732	0.24650047	0.032894	**

Table 2: UA

Comparing λ^{13} with λ^{23} in Tables 1 and 2, AA appears to be more significantly influenced by the economy-wide exogenous variables than UA since $\lambda^{13} > \lambda^{23}$. Neither AA nor UA is significantly influenced by the city-pair specific variances.

The shadow value of $q^1_{c,t}$ is $\lambda^{11}_t q^1_{c,t} + \lambda^{12}_t q^2_{c,t} + \lambda^{13}_t x^1_{t+1} + \lambda^{14}_t z^1_{t+1}$ and indicates the extent of AA's market power while $\lambda^{21}_t q^1_{c,t} + \lambda^{22}_t q^2_{c,t} + \lambda^{23}_t x^2_{t+1} + \lambda^{24}_t z^2_{t+1}$ is the shadow value of $q^2_{c,t}$ and indicates the market power of UA. Based on results from Table 1 and Table 2, we can calculate the market power of AA and UA, at sample mean values of the variables. These estimates are in Table 3.

City Pair	ORD-SAN	ORD-SEA	ORD-SFO	ORD-SLC
AA	48155.47814	42739.03961	44788.23266	44267.45226
UA	69711.50634	25440.61308	30158.55228	33775.10931

Table 3

Translating these estimates into market power shares of AA and UA in four city pairs is provided in Table 4.

From Table 4 AA has a relatively larger market power share in ORD-SEA,ORD-SFO and ORD-SLC compared with UA, especially in ORD-SEA, while AA's market share is 63%. In city-pair ORD-SAN, UA has a relatively larger market share which is 59%. We know that when there are only a few agents on one side of a market, for these agents will possess market power and the bigger the market power share, the higher profit it will have, holding other factors constant. One would expect, therefore, that UA would be more profitable in the ORD-SAN market while AA would be more profitable in the other three city pairs.

City Pair	ORD-SAN	ORD-SEA	ORD-SFO	ORD-SLC
AA	41%	63%	60%	57%
UA	59%	37%	40%	43%

Table 4: Market power share of AA and UA in city pairs

5 Summary and Conclusion

Our paper has developed a dynamic model of collusion in airport-pair routes airlines and compares the market power between the city-pairs Chicago and San Diego, Chicago and Seattle, Chicago and San Francisco and Chicago and Salt Lake City for United Airlines and American Airlines. We have used a state-space representation that is estimated by Kalman-filtering techniques to specify the first order conditions. The data we use are Databank 1A (DB1A) Department of Transportation (DOT) data. In our model, we discuss the economy-wide exogenous variables and city-pair specific variables.

In our study, we only look at all the city pairs in which American and United are dominant firms. In future work, we can extend the study to examine richer empirical settings in which, among other things, the city-pair routes are not dominated by two firms and in which exit-entry is impacted by firm conduct and market conditions. Our general framework also appears to be an appropriate and feasible vehicle for examining market conduct in other industries where merger and acquisition activities may be subject to federal antitrust authorities such as the Federal Trade Commission or administrative oversight such as the US Department of Justice.

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7 Appendix 1: Kalman filter

State space modeling is useful for the analysis of dynamic models that involve unobserved variables. The typical state model is given by Measurement Equation $y_t = Aw_t + Bx_t + u_t$ and Transition Equation $w_t = Tw_{t-1} + v_t$ where x_t is a vector of exogenous and predetermined variables, w_t is a vector of possibly unobserved state variables. u_t and v_t are uncorrelated, serially as well as crossly, with mean zero and variances R and Q, respectively. Also, the initial value w_0 is assumed to be uncorrelated with both u_t and v_t. We often assume that (u_t, v_t) is jointly normal. The Kalman Filter (1960) defines a convenient recursive procedure to compute the conditional mean and variance of the state vector w_t under normality. To introduce the filter, we first define $F_t = \sigma\left((y_s)_{s=1}^{t}\right)$ and assume that x_t is adapted to F_{t-1}. Assume normality, and introduce

$$w_{s|t} = E(w_s \mid F_t), \Omega_{s|t} = \mathrm{var}(w_s \mid F_t)$$
$$y_{s|t} = E(y_s \mid F_t), \Sigma_{s|t} = \mathrm{var}(y_s \mid F_t)$$

The Kalman filter can be effectively introduced in two steps: prediction and updating, as we will show below.

First, we write $y_t = A(T^t w_0 + T^{t-1} v_1 + \cdots + T v_{t-1} + v_t) + B x_t + u_t$ to see that $E(y_s v_t') = E(y_s u_t') = 0$ for all $s < t$. Therefore, under normality, $(y_s)_{s=1}^{t-1}$ are independent of v_t and u_t. Consequently, $E(v_t \mid F_{t-1}) = E(u_t \mid F_{t-1}) = 0$. Now, simple conditioning of the measurement and transition equations on F_{t-1} yields

$$\begin{aligned} w_{t\mid t-1} &= T w_{t-1\mid t-1} \\ y_{t\mid t-1} &= A w_{t\mid t-1} + B x_t \end{aligned}$$

with conditional variances

$$\begin{aligned} \Omega_{t\mid t-1} &= T \Omega_{t-1\mid t-1} T' + Q \\ \Sigma_{t\mid t-1} &= A \Omega_{t\mid t-1} A' + R \end{aligned}$$

as one can easily derive.

Updating involves obtaining $w_{t\mid t}$ and $\Omega_{t\mid t}$ using the values that we computed in the prediction step. Obviously, conditioning on F_t is equivalent to two-step sequential conditioning: first step conditioning on F_{t-1}, and the subsequent conditioning on y_t given F_{t-1}. Such sequential conditioning can easily be done under normality. This is because we have under normality,

$$\begin{pmatrix} w_t \\ y_t \end{pmatrix} \mid F_{t-1} \sim N\left(\begin{pmatrix} w_{t\mid t-1} \\ y_{t\mid t-1} \end{pmatrix}, \begin{pmatrix} \Omega_{t\mid t-1} & \Omega_{t\mid t-1} A' \\ A \Omega_{t\mid t-1} & \Sigma_{t\mid t-1} \end{pmatrix} \right)$$

Note that the measurement equation can be rewritten as

$$y_t - y_{t\mid t-1} = A(w_t - w_{t\mid t-1}) + u_t$$

from which we may easily see that the conditional covariance between w_t and y_t given F_{t-1} is $\Omega_{t\mid t-1} A'$. Since

$$\begin{aligned} w_{t\mid t} &= E(w_t \mid F_t) = E(w_t \mid y_t, F_{t-1}) \\ \Omega_{t\mid t} &= \mathrm{var}(w_t \mid F_t) = \mathrm{var}(w_t \mid y_t, F_{t-1}) \end{aligned}$$

we can see

$$\begin{aligned} w_{t\mid t} &= w_{t\mid t-1} + \Omega_{t\mid t-1} A' \Sigma_{t\mid t-1}^{-1} (y_t - y_{t\mid t-1}) \\ \Omega_{t\mid t} &= \Omega_{t\mid t-1} - \Omega_{t\mid t-1} A' \Sigma_{t\mid t-1}^{-1} \Omega_{t\mid t-1} \end{aligned}$$

due to If

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \right)$$

then $X_1 \mid X_2 \sim N(\mu_{1\cdot 2}, \Sigma_{11\cdot 2})$, where

$$\mu_{1\cdot 2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2)$$
$$\Sigma_{11\cdot 2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

which yield updating rules for w_t and Ω_t. In particular, the rule for w_t, $w_{t|t} - w_{t|t-1}$, is a proportion of the forecast error of y_t , $y_t - y_{t|t-1}$. The proportion is given by

$$K_t = \Omega_{t|t-1} A' \Sigma_{t|t-1}^{-1}$$

which is sometimes called the Kalman gain. The Kalman gain can thus be interpreted as the weight assigned to the information that is newly available at time t.

We have assumed thus far that the parameter values are known. They are, however, unknown and have to be estimated in most practical applications. The parameters are usually estimated by the ML method. Let θ be the vector of all the unknown parameters, and define $\ell_t(\theta)$ conditional log-likelihood function for y_t given F_{t-1}, $L_t(\theta) = \sum_{s=1}^{t} \ell_s(\theta)$ log-likelihood function for $y_{1,} \cdots y_t$ given F_0. The ML estimator of θ can be obtained by maximizing $L_t(\theta)$ with respect to θ.

It is generally not possible to obtain $\ell_t(\theta)$ explicitly as a function of θ, and we must compute it for each θ. Under normality, however, we have

$$y_t \mid F_{t-1} \sim N(y_{t|t-1}, \Sigma_{t|t-1})$$

with our notation, and therefore $\ell_t(\theta)$ can easily be calculated as

$$\ell_t(\theta) = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log\det\Sigma_{t|t-1} - \frac{1}{2}\left(y_t - y_{t|t-1}\right)' \Sigma_{t|t-1}^{-1}(y_t - y_{t|t-1})$$

Note that $y_{t|t-1}$ and $\Sigma_{t|t-1}$ are functions of θ, and that they are computed in the prediction step of the Kalman filter for each θ.

We are often interested in the forecasts and their variances of w_t given all the observations of $(y_t)_{t=1}^{n}$. To obtain $w_{t|n} = E(w_t \mid F_n)$ and $\Omega_{t|n} = \mathrm{var}(w_t \mid F_n)$, we first observe that

$$E\left(w_t \mid w_{t+1,} F_n\right) = E\left(w_t \mid w_{t+1}, F_t\right)$$

It can be easily seen from

$$y_{t+k} = A(T^{k-1} w_{t+1} + T^{K-2} v_{t+2} + \cdots + v_{t+k}) + Bx_{t+k} + u_{t+k}$$

that y_{t+k} is uncorrelated with, and therefore under normality, independent of w_t given w_{t+1} and F_t , for $k = 1, 2, \ldots$. However, since

$$w_{t+1} - w_{t+1\,|\,t} = T(w_t - w_{t\,|\,t}) + v_{t+1}$$

we have $\operatorname{cov}(w_t, w_{t+1} \mid F_t) = \Omega_{t\,|\,t} T'$ due to

$$\begin{pmatrix} w_t \\ w_{t+1} \end{pmatrix} \mid F_t \sim N\left(\begin{pmatrix} w_{t\,|\,t} \\ w_{t+1\,|\,t} \end{pmatrix}, \begin{pmatrix} \Omega_{t\,|\,t} & \Omega_{t\,|\,t} T' \\ T\Omega_{t\,|\,t} & \Omega_{t+1\,|\,t} \end{pmatrix}\right)$$

therefore

$$E\left(w_t \mid w_{t+1}, F_t\right) = w_{t\,|\,t} + \Omega_{t\,|\,t} T' \Omega_{t+1\,|\,t}^{-1} (w_{t+1} - w_{t+1\,|\,t})$$

Thus, we know

$$w_{t\,|\,n} = E(E\left(w_t \mid w_{t+1,} F_n\right) \mid F_n) = w_{t\,|\,t} + \Omega_{t\,|\,t} T' \Omega_{t+1\,|\,t}^{-1} (w_{t+1\,|\,n} - w_{t+1\,|\,t})$$

To obtain $\Omega_{t\,|\,n}$, we first get

$$w_{t\,|\,n} - w_{t\,|\,t} = \Omega_{t\,|\,t} T' \Omega_{t+1\,|\,t}^{-1} (w_{t+1\,|\,n} - w_{t+1\,|\,t})$$

We also know

$$E\left(w_{s\,|\,n} \mid F_t\right) = E\left\{E\left(w_s \mid F_n\right) \mid F_t\right\} = E\left(w_s \mid F_t\right) = w_{s\,|\,t}$$

due to $F_t \subset F_n$ .Therefore, we have

$$E\left(w_{s\,|\,n} - w_{s\,|\,t}\right)\left(w_{s\,|\,n} - w_{s\,|\,t}\right)' = Ew_{s\,|\,n} w_{s\,|\,n}' - Ew_{s\,|\,t} w_{s\,|\,t}'$$

Moreover, by the same token,

$$Ew_{s\,|\,t} w_{s\,|\,t}' = Ew_s w_s' - E\left(w_s - w_{s\,|\,t}\right)\left(w_s - w_{s\,|\,t}\right)' = Ew_s w_s' - \Omega_{s\,|\,t}$$

We may now easily deduce

$$\Omega_{t|n} = \Omega_{t|t} + \Omega_{t|t} T' \Omega_{t+1|t}^{-1} (\Omega_{t+1|n} - \Omega_{t+1|t}) \Omega_{t+1|t}^{-1'} T \Omega_{t|t}'$$

which can be used to get $\Omega_{t|n}$ in a successive manner.

8 Appendix 2: Shadow price determination

Shadow price calculation is an important by produce of the dynamic program and is useful in analyzing equilibrium strategies and market power. Let's first look at a simplest optimization problem – two variables and one equality constraint:

$$\max f(x, y)$$

subject to

$$h(x, y) = a$$

Let f and h be C^1 functions of two variables. For any fixed value of the parameter a, let $(x^*(a), y^*(a))$ be the solution of the above problem with corresponding multiplier $\mu^*(a)$. Suppose that $x^*, y^*,$ and μ^* are C^1 functions of a and that the nondegenerate constraint qualification (NDCQ) holds at $(x^*(a), y^*(a), \mu^*(a))$. Then

$$\mu^*(a) = \frac{d}{da} f(x^*(a), y^*(a))$$

The Lagrangian for this problem is

$$L(x, y, \mu; a) \equiv f(x, y) - \mu(h(x, y) - a)$$

The optimal solutions for $(x^*(a), y^*(a), \mu^*(a))$ must satisfy

$$0 = \frac{\partial L}{\partial x}(x^*(a), y^*(a), \mu^*(a); a) = \frac{\partial f}{\partial x}(x^*(a), y^*(a), \mu^*(a)) - \mu^*(a)\frac{\partial h}{\partial x}(x^*(a), y^*(a), \mu^*(a))$$
$$0 = \frac{\partial L}{\partial y}(x^*(a), y^*(a), \mu^*(a); a) = \frac{\partial f}{\partial y}(x^*(a), y^*(a), \mu^*(a)) - \mu^*(a)\frac{\partial h}{\partial y}(x^*(a), y^*(a), \mu^*(a))$$
$$h(x^*(a), y^*(a)) = a$$

for all a. Since $h(x^*(a), y^*(a)) = a$ it must be that

$$\frac{\partial h}{\partial x}(x^*, y^*)\frac{dx^*}{da}(a) + \frac{\partial h}{\partial y}(x^*, y^*)\frac{dy^*}{da}(a) = 1$$

for every a . Therefore, using the Chain Rule,

$$\begin{aligned}\frac{d}{da} f(x^*(a), y^*(a)) &= \frac{\partial f}{\partial x}(x^*(a), y^*(a))\frac{dx^*}{da}(a) + \frac{\partial f}{\partial y}(x^*(a), y^*(a))\frac{dy^*}{da}(a) \\ &= \mu^*(a)\frac{\partial h}{\partial x}(x^*(a), y^*(a))\frac{dx^*}{da}(a) + \mu^*(a)\frac{\partial h}{\partial y}(x^*(a), y^*(a), \mu^*(a))\frac{dy^*}{da}(a) \\ &= \mu^*(a)[\frac{\partial h}{\partial x}(x^*(a), y^*(a))\frac{dx^*}{da}(a) + \frac{\partial h}{\partial y}(x^*(a), y^*(a), \mu^*(a))\frac{dy^*}{da}(a)] \\ &= \mu^*(a)\cdot 1\end{aligned}$$

and $\mu^*(a)$ measures the rate of change of the optimal value of f with respect to the parameter a. It is not hard to extend the above to the setting below.

Let f, $h_1, \ldots, h_m$ be C^1 function on $\mathbf{R}^n$. Let $\mathbf{b} = (b_1,...,b_m), \mathbf{c} = (c_1,...,c_k)$ be exogenous parameters. Consider the problem of maximizing $f(x_1,..., x_n)$ subject to the constraints

$$h_1(x_1,..., x_n) = b_1,...h_m(x_{1,}..., x_n) = b_m$$

and inequality constraints

$$g_1(x_1,..., x_n) \le c_1,...g_k(x_{1,}..., x_n) \le c_k$$

Let $x_1^*,..., x_n^*$ denote the solution of this problem, with corresponding Lagrange multipliers $\mu_1^*(\mathbf{b}),..., \mu_m^*(\mathbf{b}), \lambda_1^*(\mathbf{c}),...\lambda_k^*(\mathbf{c})$. Suppose further that the x_i^* 's are differentiable function of $(b_1,...,b_m, c_1,...,c_k)$, μ_j^*'s and λ_s^* are differentiable function of $(b_1,...,b_m)$ and $(c_1,...,c_k)$, respectively, and NDCQ holds. Then for each $j = 1,...m$

$$\mu_j^*(b_1,...b_m) = \frac{\partial}{\partial b_j} f(x_1^*(b_{1,}..b_m, c_1,...c_k),...x_n^*(b_{1,}..b_m, c_1,...c_k))$$

and for each $s = 1,..., k$

$$\lambda_s^*(c_1,...c_k) = \frac{\partial}{\partial c_s} f(x_1^*(b_{1,}..b_m, c_1,...c_k),...x_n^*(b_{1,}..b_m, c_1,...c_k))$$

We can specify the objective function $f(\mathbf{x})$ as the profit function of a firm and interpret the a_j's on the right-hand sides of the constraints as representing the amounts available for inputs in the firm's production process. In this situation,

$$\frac{\partial}{\partial a_j} f(x_1^*(\mathbf{a}),...x_n^*(\mathbf{a}))$$

represents the change in the optimal profit resulting from the availability of one more unit of input *j* and indicates how valuable another unit of input *j* would be to the firm's profits. Alternatively, it tells the maximum amount the firm would be willing to pay to acquire another unit of input *j*. For this reason, $\lambda_j^*(\mathbf{a})$ is called the shadow price of input *j*.